AIAA JOURNAL
Vol. 30, No. 11, November 1992

Nonlinear Vibration and Radiation from a Panel with Transition to Chaos

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The dynamic response of an aircraft panel forced at resonance and off-resonance by plane acoustic waves at normal incidence is investigated experimentally and numerically. Linear, nonlinear (period doubling), and chaotic responses are obtained by increasing the sound pressure level of the excitation. The response time history is sensitive to the input level and to the frequency of excitation. The change in response behavior is due to a change in input conditions, triggered either naturally or by modulation of the bandwidth of the incident waves. Off-resonance bifurcation is diffused and difficult to maintain; thus the panel response drifts into a linear behavior. The acoustic pressure emanated by the panel is either linear or nonlinear as is the vibration response. The nonlinear effects accumulate during the propagation with distance. Results are also obtained on the control of the panel response using damping tape on an aluminum panel and a graphite epoxy panel having the same size and weight. Good agreement is obtained between the experimental and numerical results.

I. Introduction

IT is often required in the structural dynamic analysis of flight vehicles to consider the interaction of sound and flow with the flexible structure. The traditional procedure is to analyze the response of the structure and assume that the vibration is linear and stationary. However, certain experimental results indicate that the response of a structure can be both nonstationary and nonlinear.[1] A typical example is the response of the structure of a space vehicle during an accelerated or decelerated flight. Similarly, transport aircraft fuselage panels in the vicinity of an engine inlet/outlet or a propeller are known to resonate. These examples have stimulated the search for universal features describing nonlinear systems with some analogy between measurements and mathematical models.[2-9] Unlike the linear response, the nonlinear one is sensitive to changes in input conditions. Dowell and Pezeshki[10] studied the dynamics of a buckled beam and found that chaos was not difficult to obtain. They also investigated the problem of initial condition and external excitation. Both in flight and in a wind tunnel, the input conditions vary continuously because of, for example, vibration, noise, and atmospheric perturbations, but most of all because of changes in airplane speed. Thus, one can raise the question about the accuracy of the traditional procedure in predicting the response of structures in the nonlinear regime.[11-13] In the 1960s, the nonlinear theory of elastic waves did not receive much attention, although it was recognized that linearized equations provide no more than a first approximation to actual situations. Currently, it is widely accepted that linear theory is inadequate in explaining certain phenomena, such as chaos; thus a nonlinear approach is required. However, the importance of nonlinear analysis of model equations can be overestimated because of the absence of acoustic coupling in most models.

The problem of nonlinear wave propagation in a medium has been examined by several authors.[2,5,6,14,15] The bifurcation in the context of acoustic chaos has been introduced by Lauterborn and Cramer.[14] Various methods have been used in the analysis, including multiple scale, averaging, and perturbation. Results have indicated a cumulative growth of nonlinear effects resulting from the excitation of the panel interfacing with an acoustic fluid. As the distance from the surface increases, nonlinear distortion of the linear acoustic wave and shock formation can occur.

In this paper, we focus our attention on the nonlinear vibration and nonlinear acoustic radiation of a typical aircraft fuselage panel forced by pure tone, incident plane acoustic waves. The investigation is conducted both experimentally and numerically. This study is motivated by the results obtained by Maestrello and Grosveld,[16] in which a typical aircraft panel was forced by a boundary-layer flow with a superimposed acoustic excitation. These results indicated that at times the panel responded nonlinearly to the forcing field. The present experiments are undertaken to enhance our understanding of nonlinear responses and simple chaotic motion of a panel before complex experiments involving random convective input loads are conducted.

Numerically, the nonlinear plate equations are integrated using a method developed by Robinson.[17] The panel is forced by plane waves at normal incidence. It is important to mention that, as a result of the nonlinear vibration behavior of the surface, acoustic waves are emitted from the surface and propagate away as nonlinear acoustic waves.[14,15,18]

The rest of the paper is organized as follows. The experimental setup and procedure are described in Sec. II, and the mathematical model describing the physical situation along with the numerical technique used are given in Sec. III. Section IV is divided into several subsections, each one describing a set of experimental results. The numerical results are given in Sec. V. A summary of the results and the concluding remarks are presented in Sec. VI.

Received March 3, 1992; revision received May 8, 1992; accepted for publication May 21, 1992.

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II. Experimental Setup

An experiment is set up to study the linear and nonlinear response of panels excited by plane acoustic waves at normal incidence. Two clamped rectangular panels of equal weight, one made of aluminum and the other of a composite material, having sizes of $12 \times 8 \times 0.040$ and $12 \times 8 \times 0.0736$ in., respectively, are used in separate experimental tests. Each clamped panel is mounted in a rigid absorbing partition dividing two anechoic rooms, the source and the transmission rooms (see Fig. 1). The composite panel is constructed of 14 layers of graphite epoxy with alternating orientation between $\pm 45/0/90$ and $\mp 45/0/90$ deg. The normal incidence plane waves are generated in the source room by an acoustic driver mounted at one end of an exponential horn. A pressure transducer is mounted in the exponential horn, flush with the inner surface to monitor the speaker output pressure. The transmitted pressure wave and panel vibration are measured in the transmission room. The vibration of the plate is measured by strain gauges located at the midpoint of the top and the side edges, and the transmitted wave is measured by two unvented pressure transducers.

III. Formulation of the Model

The plate motion is described by a system of three nonlinear partial differential equations given by[19]

$$D \nabla^4 w + \rho h w_{tt} + \gamma w_t = (p_1 - p_2) + \frac{Eh}{1-\nu^2}\,[(u_x^0 + \tfrac{1}{2} w_x^2)(w_{xx} + \nu w_{yy}) + (v_y^0 + \tfrac{1}{2} w_y^2)(w_{yy} + \nu w_{xx}) + (1-\nu) w_{xy}(u_y^0 + v_x^0 + w_x w_y)] \tag{1}$$

$$u_{xx}^0 + d_1 u_{yy}^0 + d_2 v_{xy}^0 = -w_x(w_{xx} + d_1 w_{yy}) - d_2 w_y w_{xy}$$

$$v_{yy}^0 + d_1 v_{xx}^0 + d_2 u_{xy}^0 = -w_y(w_{yy} + d_1 w_{xx}) - d_2 w_x w_{xy}$$

where

$$\nabla^4 w = w_{xxxx} + 2w_{xxyy} + w_{yyyy} \tag{2}$$

$$d_1 = \frac{1-\nu}{2}, \qquad d_2 = \frac{1+\nu}{2}, \qquad D = \frac{Eh^2}{12(1-\nu^2)} \tag{3}$$

and where u^0 and v^0 are the in-plane displacements and w the out-of-plane displacement. The physical constants in Eqs. (1) are the stiffness D, the density ρ, the plate thickness h, the physical damping γ, the modulus of elasticity E, and the Poisson ratio of the material ν. The various subscripts involving the variables (x, y, t) indicate partial derivatives. The two pressure terms on the right-hand side of the first equation represent the incident and reflected wave from a rigid wall and the waves generated by the panel oscillations. These two terms are

$$p_1 = 2p_0 \sin(\omega t)$$
$$p_2 = 2p_{\text{Euler}} \tag{4}$$

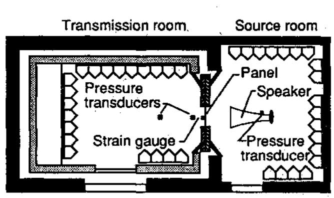

Fig. 1 Experimental setup for measurements of strain and sound radiation from a panel.

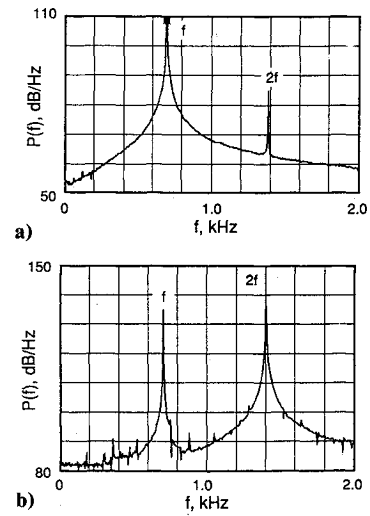

Fig. 2 Power spectral density of the input pressure: a) linear, b) nonlinear.

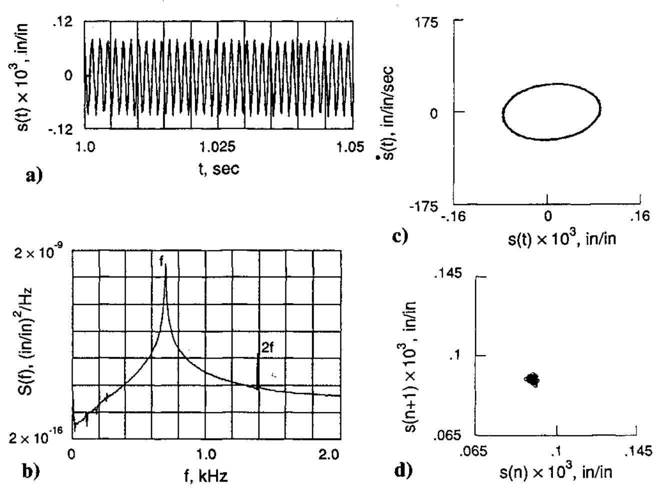

Fig. 3 Linear strain response: a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

where $p_0 \sin(\omega t)$ is the incident wave, and p_{Euler} represents the transmitted wave, which is obtained by integration of the nonlinear Euler equation coupled to the plate equations. Because of the rigidity of the plate, we assume that the incident wave is much larger than the transmitted wave, $p_{\text{Euler}} \ll p_0$. At the present time, we uncouple the plate motion from the fluid motion by omitting p_{Euler} in Eq. (1). This problem will be analyzed in a future work, and therefore we will obtain a more realistic solution to the problem.

For a given frequency f, the amplitude p_0 is varied from 100 to 130 dB. The set of partial differential equations given earlier are subject to initial and boundary conditions

$$\begin{aligned} t &= 0 \qquad & u^0 = v^0 = w = 0 \\ x &= 0, a \qquad & u^0 = v^0 = w = w_x = w_y = 0 \\ y &= 0, b \qquad & u^0 = v^0 = w = w_y = w_x = 0 \end{aligned} \tag{5}$$

representing a rigidly clamped plate initially at rest. The system of equations is integrated numerically using a finite element method developed by Robinson.[17] The plate dimensions are the same as those used in the experiments.

IV. Experimental Results

The excitation frequency $f = 0.7$ kHz used in the experiments corresponds to one of the panel's resonant frequencies, and the sound pressure level inside the exponential horn is set at 109 and 135 dB. These levels are chosen such that the panel response is linear at the lower sound pressure level and nonlinear at the higher level. The pressure power spectral densities $P(f)$ are shown in Figs. 2a and 2b. The waves inside the exponential horn travel in a direction normal to the panel surface as a simple linear plane wave (see Fig. 2a) and as a wave with higher harmonics (see Fig. 2b).

A. Linear Response

The strain response to the acoustic excitation with input spectral density of Fig. 2a is shown in Fig. 3. The strain is measured at the midpoint of the long edge of the plate. The time history of the response, $s(t)$ shown in Fig. 3a, shows a regular and repetitive pattern indicative of a linear response. The corresponding power spectrum $S(f)$ shows a strong peak at the fundamental frequency and a much weaker peak for the harmonic (see Fig. 3b). Both the phase diagram of the strain, as shown in Fig. 3c [$\dot{s}(t)$ vs $s(t)$, where the ($\dot{}$) indicates the time derivative], and the Poincaré map, as shown in Fig. 3d [$s(n + 1)$ vs $s(n)$, where n indicates a running index], are typical of a linear response as indicated by the trajectories passing through a single point. This means that the response does not change with time or that it has reached an equilibrium with the forcing field.

B. Nonlinear Response

The response of the panel increases in amplitude and in bandwidth as a result of increasing the input load or the incident wave amplitude. The change from linear to nonlinear response is attributed to the increment of the load at a resonant frequency over a threshold value. After the increment, the changeover occurs with a time delay that is sensitive to the initial condition and the rate of the load increment. Once the nonlinear response is triggered, the oscillations evolve through a subharmonic formation known as *period doubling* bifurcation as shown in Fig. 4. In this type of bifurcation, a limit cycle of a given period changes to a limit cycle of exactly double the period, and after a finite cascade of period doublings, a chaotic attractor can be observed. This type of nonlinear phenomenon was among the first observed in an acoustic experiment by Lauterborn and Cramer[14] in which they demonstrated the existence of chaotic behavior. This problem will be further discussed in the next section.

The time history of the strain $s(t)$ shows that in each forcing period there are four peaks corresponding to the subharmonics (see Fig. 4a). The power spectrum of the response, $S(f)$ in Fig. 4b, shows the period doubling route from the appearance of peaks at the halves of the fundamental and several higher harmonics, e.g., $f/4, f/2$, and $3f/2$. The phase diagram shows four different orbits in Fig. 4c that become four points on the Poincaré map as shown in Fig. 4d. Each of these orbits in the phase plane corresponds to either the fundamental frequency or its subharmonics. It shows that the trajectory in the phase plane is periodic. To arrive at this final state of bifurcation, the motion remains linear in the early stage, then the first trigger produces the first subharmonic $f/2$, and the subsequent one develops the second subharmonic $f/4$. Once the nonlinear response is established, the process is not reversed by a decrease in the input pressure level. For example, the nonlinear response is still maintained when the input pressure is reduced by 20 dB below the maximum excitation level. We have described here an example of nonlinear response, with the driving force at a resonant frequency, to identify the deterministic events before the chaotic motion.

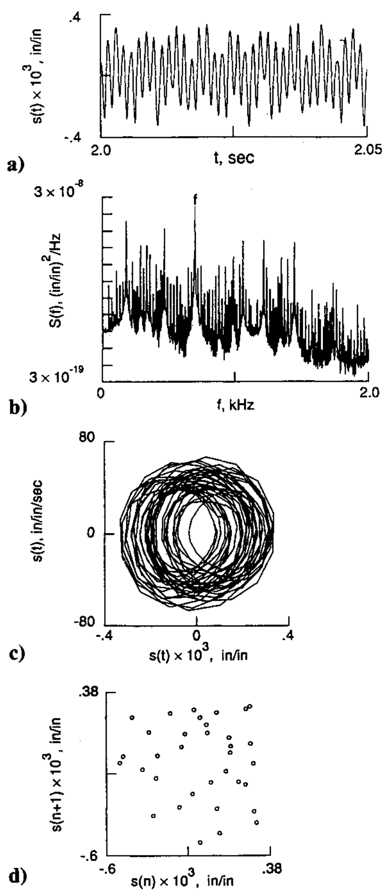

Fig. 5 Chaotic strain response: a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

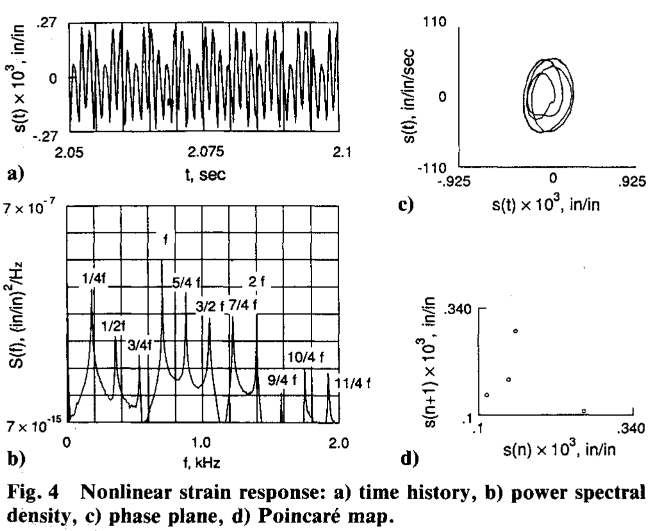

Fig. 4 Nonlinear strain response: a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

C. Chaotic Response

As a result of further increasing the incident acoustic pressure level, the number of bifurcations increases; thus the transition takes the system into a chaotic regime characterized by a broadband spectrum. An example of such a response is shown in Fig. 5. The time history of the strain $s(t)$ shows the nonperiodic character of the response (see Fig. 5a), supported by the broadband nature of the power spectrum $S(f)$ in Fig. 5b. Only a few narrow peaks can be recognized in the spectrum. The corresponding motion in the phase plane is shown in Fig. 5c. The trajectories are chaotic, exhibiting many indistinguishable orbits that never settle down to a periodic behavior. This chaotic sequence creates a random phase modulation that broadens the power spectrum. The route from linear to chaotic response is through subharmonic (½, ¼) formation. After a few bifurcations, the peaks become steeper as can be seen by comparing Figs. 4b and 5b. This phenomenon was previously observed by other investigators.[14] As the input level increases even further, new bifurcations occur from which new chaotic states emerge. Physically, this sequence reflects the fact that the large input level makes the panel motion gain enough energy to exhibit increasingly nonlinear effects. The phase diagram shows that the response is highly irregular; there are no distinct orbits in Fig. 5c. Now the Poincaré map is made up of a set of scattered points, showing the characteristic of a chaotic response. It is important to note that in our experiment the chaotic state is triggered at one resonant mode that demonstrates one of the routes to chaos.

A useful dynamical diagnostic for chaotic systems is the Lyapunov exponent. Any system containing at least one positive Lyapunov exponent is known to be chaotic. To further confirm our conclusion, the Lyapunov exponent for a time series data sample (composed of 15,000 points) is calculated using a method outlined by Zeng et al.,[20] and the Lyapunov exponent is found to be equal to + 10, which is characteristic of a highly chaotic state.

D. Off-Resonant Excitation

Different types of oscillations appear in panel responses when the input level is kept fixed while the driving frequency is changed from resonant to off-resonant. In this experiment, the new states correspond to a change from a resonant frequency of 0.7 kHz to an off-resonant frequency of 0.685 kHz. The strain power spectral density is shown in Fig. 5b for resonant and in Fig. 6c for off-resonant frequencies. Figures 6a and 6b are for intermediate states corresponding to the frequencies 0.695 and 0.690 kHz. The power spectrum for the 0.7 kHz excitation is broadband. As a result of decreasing the frequency by 5 Hz to 0.695 kHz, the spectrum shows discrete peaks rather than the broadband response in Fig. 6a. By further decreasing the forcing frequency to 0.690 kHz, the power spectral density approaches that of the linear response (Fig. 6b). This process continues until the power spectral density resembles that of the linear response at a frequency of

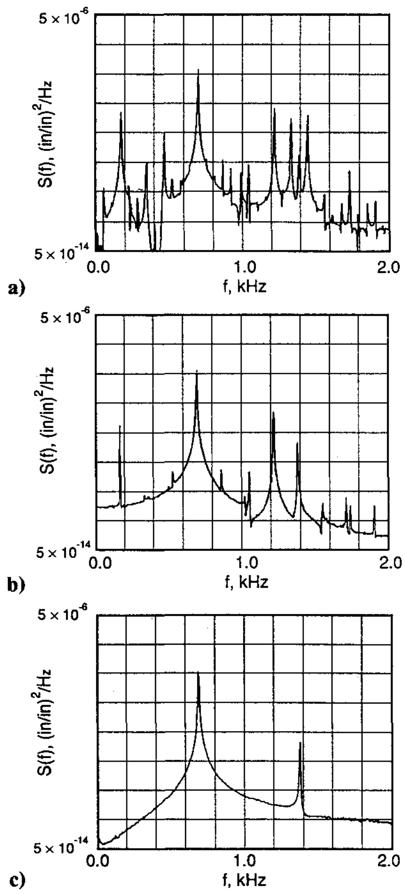

Fig. 6 Strain power spectral density at constant input for different frequencies: a) 0.695 kHz, b) 0.690 kHz, c) 0.685 kHz.

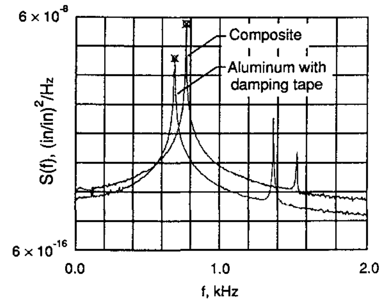

Fig. 7 Strain power spectral density for an aluminum panel with damping tape and for a composite panel.

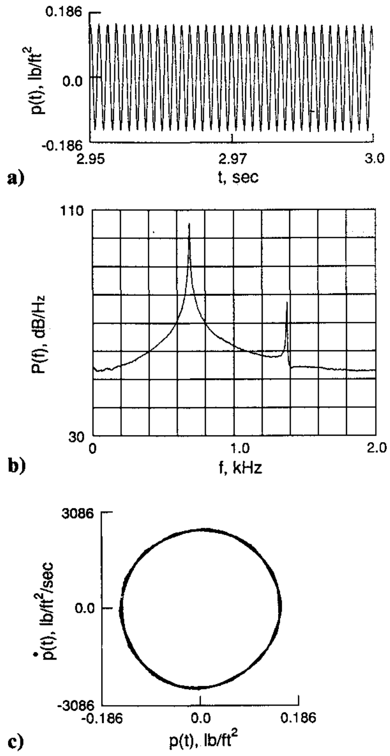

Fig. 8 Radiated pressure 2 in. from the panel (linear): a) time history, b) power spectral density, c) phase plane.

0.685 kHz in Fig. 6c. Therefore, a panel in a chaotic vibration state can be detuned by slightly offsetting the forcing frequency or, equivalently, by altering the structure's natural frequencies.

E. Control of Chaotic Response and the Role of Composite Structure in Delaying Nonlinear Response

The need to improve sonic fatigue resistance of structures has become important because of the demand to increase the lifetime of the structure. As a result, a method is proposed whereby a nonlinear chaotic motion of a panel can be reduced to a linear motion by a small change in the structural parameters. This is achieved by the use of aluminum foil damping tape. It has been experimentally demonstrated that a damping tape is effective in the frequency range between 0.8–5 kHz.[21] Results show the change in panel response, from a nonlinear to a linear one, by adding damping tape to the surface (see Fig. 7) as compared with that of a panel without damping tape (see Fig. 5b). The addition of the tape changes the natural frequency but also increases the damping characteristic of the panel. When the modified panel is forced at its natural frequency, with the same input loading, the response remains linear. This result demonstrates that damping tape can change the dynamics of the motion when excited at resonance. There-

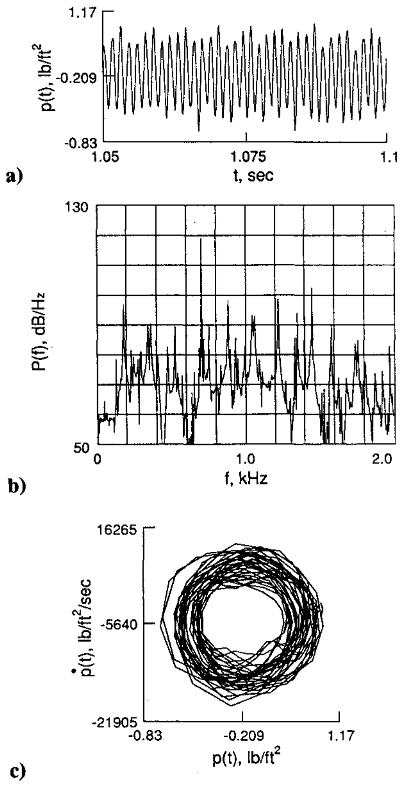

Fig. 9 Radiated pressure 2 in. from the panel (nonlinear): a) time history, b) power spectral density, c) phase plane.

fore, chaos can be delayed and the linear response can be maintained.

Composite materials have a great potential in reducing the acoustic induced vibration. The most attractive properties are the high strength-to-weight and stiffness-to-weight ratios. These enhance the excellent fatigue strength, ease of formability, wide range of operating temperatures, high damping, and resistance to corrosion. A composite panel having the same size and weight as the aluminum panel is forced at resonance by the same acoustic pressure loading. The response power spectral density of the strain is shown in Fig. 7. The resonant frequency of the composite panel is 0.77 kHz, which is higher than the aluminum panel. The excitation frequency is also changed from 0.7 to 0.77 kHz. Whereas the response of the undamped aluminum panel is nonlinear, the response of the composite panel remains linear. This indicates that at resonance the linear response range or the threshold value of the load for the composite panel is larger than that for the aluminum panel.

F. Linear and Nonlinear Acoustic Radiation from the Panel Response

Acoustic measurements are made in the transmission room at 2 in. and at 4 ft from the center of the panel. The time history of the acoustic pressure, the power spectral density, and the phase diagram corresponding to the linear and nonlinear pressure are shown in Figs. 8 and 9 for the measurements taken at 2 in. from the center of the panel forced by the linear and nonlinear loads of Fig. 2. The time history of the radiated acoustic pressure, Fig. 8a, shows the periodic nature of the signal indicative of a linear behavior, which is consistent with the strain response of Fig. 3a. The results in Fig. 4a for the strain and Fig. 9a for the acoustic pressure show that both are nonlinear. A similar conclusion can be made for the power spectral density and the phase diagrams. It is noted that the nonlinearity in the panel vibration results in a nonlinearity in the radiated acoustic pressure. In the linear response, the surface interfacing with the surrounding fluid is not coupled to the field that propagates because the acoustic radiation is weak and is over a narrowband, pure tone. When the panel vibration is nonlinear or chaotic, the vibration gives rise to noticeable sound emission over a broadband. Thus, the motion of the panel may produce acoustic chaos (see Fig. 9). As the wave travels away from the surface, it undergoes a progressive distortion until a point is reached at which a shock front may appear. At 4 ft from the center of the panel, the pressure shown in Fig. 10 is nonlinear. The time history of the propagation front becomes steeper and would eventually give rise to a weak shock. Since the present nonlinear level is weak, a shock may not form. The power spectrum shows that the high frequencies have higher amplitude than the lower frequencies, indicating broadening of the spectrum in comparison with Fig. 9b. Similar observations can be made on the time histories and the phase diagrams. The Poincaré map also indicates that the sound field reaching the pressure transducer is chaotic.

V. Numerical Results

The results obtained by numerical integration of Eqs. (1) and (5) for a fixed forcing frequency $f = 0.754$ kHz (which is a resonant frequency of the given plate) and variable amplitude p_0 (from 100 to 130 dB) are shown in Figs. 11–13. Each of these figures is composed of four different plots for the center plate displacement and velocity: a) time history b) power spectral density, c) phase diagram, and d) the Poincaré map. Figure 11 shows the plate response for an incident pressure level of 100 dB. The power spectrum of the displacement shows only the fundamental frequency that is in agreement

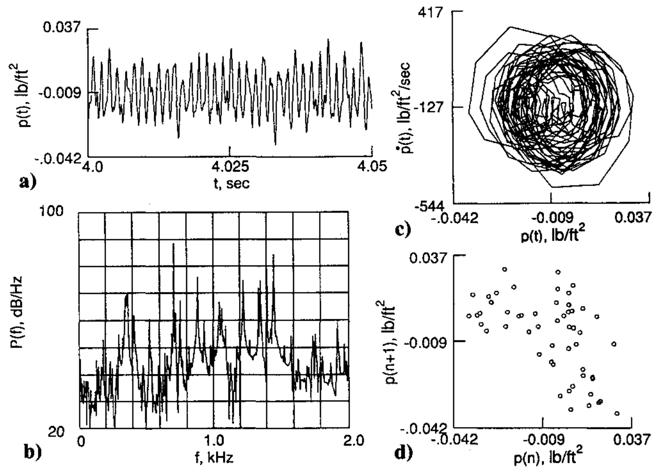

Fig. 10 Radiated pressure 4 ft from the panel (nonlinear): a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

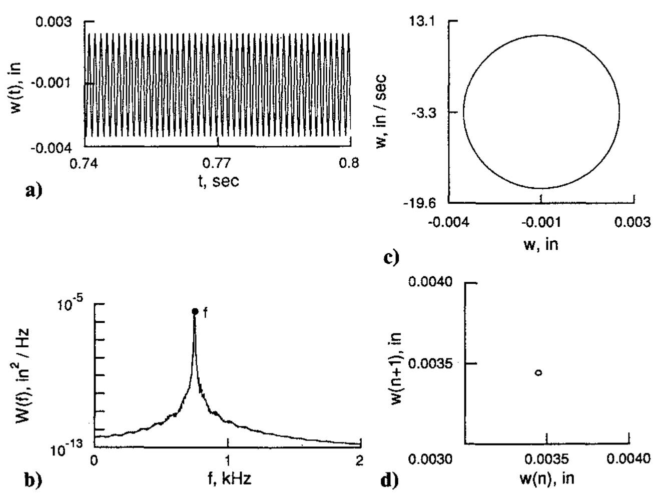

Fig. 11 Linear displacement response (numerical): a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

with the experimental results. The phase diagram shows a circle and the corresponding Poincaré map is a point. This type of behavior is characteristic of a linear response.

Increasing the incident pressure level to 127 dB leads to a change in the plate response. This change is characterized by the presence of harmonics and subharmonics in the power spectrum. As the pressure level is increased, first a higher harmonic is obtained, and a further increase of the level leads to the first bifurcation by the formation of a subharmonic ($f/2$). This subharmonic creates its own higher harmonics, and by further increasing the excitation level, a second bifurcation occurs ($f/4$). Figure 12b shows the existence of three subharmonics ($f/4$, $f/2$ and $3f/4$) and six more higher harmonics ($5f/4$, $3f/2$, $7f/2$, $2f$, $9f/4$, and $5f/2$). Similarly, the time history of the displacement clearly shows a fundamental frequency with three subharmonics (see Fig. 12a). This type of bifurcation behavior is known as period doubling; each peak has a period of four. The fundamental and subharmonic frequencies translate in the phase plane to four different orbits (one orbit for each frequency, see Fig. 12c), and the corresponding Poincaré map shows four points (see Fig. 12d). The behavior shown in Fig. 12 is characteristic of a nonlinear response and is consistent with the experimental results.

The process of bifurcation continues as the input pressure level is increased, and at a high enough input level the various peaks in the power spectrum become indistinguishable, which is in agreement with the experimental results. However, quantitative comparisons with experiments have not been made

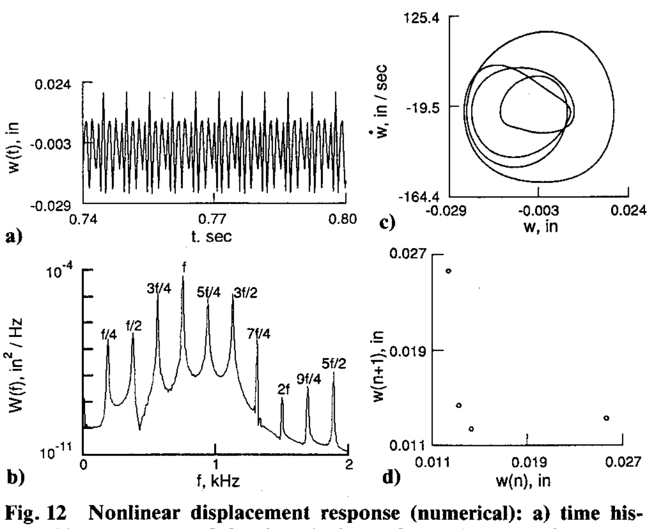

Fig. 12 Nonlinear displacement response (numerical): a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

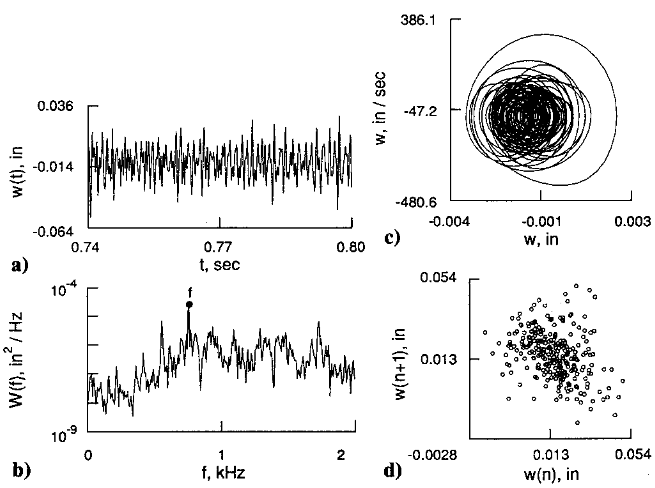

Fig. 13 Chaotic displacement response (numerical): a) time history, b) power spectral density, c) phase plane, d) Poincaré map.

because one has to account for the acoustic damping. This type of behavior is shown in Fig. 13 for a pressure level of 130 dB. The only peak in the power spectrum that can be recognized is that of the fundamental frequency f. The difference in level between the fundamental and the others is small, and in this situation the power spectrum is called a broadband spectrum. The time history of the displacement shows no periodicity of the signal. This is confirmed by both the phase diagram and the Poincaré map. No dominant single orbit is observed on the phase diagram, and the Poincaré map shows a large number of points. This describes a highly nonlinear behavior generally known as a chaotic response. This conclusion is further supported by the calculation of the Lyapunov exponent, which is found to be equal to + 11.

VI. Discussion and Conclusions

In this paper, attention is focused on the effects of acoustic loading at normal incidence on the stability of a panel structure vibration and on the resultant near- and far-field acoustic pressure as the response changes from a linear to a chaotic one. Depending on the amplitude and frequency of the excitation, the response of the panel undergoes qualitative changes and settles down to periodic oscillations or yields chaotic oscillations as a final state of nonlinearity. The motion normally starts periodically. As the input sound pressure level increases, it becomes more difficult to recognize the original deterministic oscillations since the system drifts from a nonlinear to a chaotic state with time. The route to chaos is preceded by the formation of subharmonics, a nonlinear response behavior. The period doubling route to chaos can best be observed in the power spectrum of the data, both experimental and numerical results, with the successive appearance of subharmonics and harmonics as the process evolves. Although nonlinear response generally occurs at a sound pressure level of about 135 dB, experimental results also indicate that it could be maintained as low as 110 dB once the nonlinear response was triggered. This is believed to be due to the influence of the hysteretic damping.

Because of the increment in the damping and stiffness characteristics, both a graphite epoxy laminated panel and an aluminum panel with damping tape are found to perform better than the conventional aluminum panel when forced by acoustic waves at a resonant frequency.

The numerical results showed that, when a panel is excited at a resonant frequency by plane acoustic waves, linear, nonlinear, and chaotic responses can be obtained by changing the intensity of the loading. These results are in good qualitative agreement with the experiments.

Acknowledgments

The second and third authors were supported by NASA Langley Research Center under Contracts NAS1-19317 and NAS1-19000, respectively.

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